THERMODYNAMICS OF LATTICE QCD WITH 2 QUARK FLAVOURS: CHIRAL SYMMETRY AND TOPOLOGY. [a]

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We have studied the restoration of chiral symmetry in lattice QCD at the finite temperature transition from hadronic matter to a quark-gluon plasma. By measuring the screening masses of flavour singlet and non-singlet meson excitations, we have seen evidence that, although flavour chiral symmetry is restored at this transition, flavour singlet $(U(1))$ axial symmetry is not. We conclude that this indicates that instantons continue to play an important rôle in the quark-gluon plasma phase.

1 Introduction

QCD thermodynamics describes hadronic/nuclear matter at finite temperature and/or baryon number density. This is relevant to the early universe, neutron stars and relativistic heavy ion collisions (RHIC).

Hadronic matter undergoes a transition to a quark-gluon plasma at a temperature $T = T_c \approx 100$–150 MeV [1,2]. This transition is believed to be a second order phase transition when $m_u, m_d \to 0$ (m_s appears to be too large to make this transition first order). From now on we shall ignore the s quark.

At this phase transition the $SU(2) \times SU(2)$ chiral flavour symmetry is restored. This means that in the plasma phase the effective (screening) masses of the π and $\sigma(f_0)$ become equal as do the masses of the η/η' and $\delta(a_0)$. The question remains as to whether the anomalous $U(1)_{axial}$ symmetry is also restored at this transition [3,4] If so, all 4 screening masses would become equal above the transition. The rest of this talk presents lattice measurements aimed at resolving this question, and discussion of how this is related to topological charge and fermion zero modes.

If the $U(1)_{axial}$ symmetry is not restored at the chiral(deconfinement) transition, this means that instantons still play an important rôle in the plasma phase.

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[a]talk presented by D. K. Sinclair at Workshop on Nonperturbative Methods in Quantum Field Theory, 2nd – 13th February, 1998, Adelaide, South Australia.

The functional integral for a field theory at Euclidean time in a region which is infinite in each of the spatial directions but has a finite extent, $1/T$, in the time direction with periodic (antiperiodic) boundary conditions, is the canonical partition function for the field theory at temperature T. This allows us to simulate lattice QCD at finite T by simulating it on a lattice whose spatial dimensions are much larger than its time dimension.

This talk presents work which is reported in detail in [5], which should be consulted for a more complete description and for more complete references. For related work which draws similar conclusions see [6,7].

2 Meson propagators and Topological Charge at finite Temperature

In QCD, mesons can be created or annihilated by operators quadratic in the quark fields. Thus a meson propagator has the form

$$G(x, y) = \langle \bar{\psi}(x) \mathcal{O}^\dagger \psi(x) \bar{\psi}(y) \mathcal{O} \psi(y) \rangle. \tag{1}$$

For the π, $\mathcal{O} = \gamma_5 \tau$, for the σ, $\mathcal{O} = 1$, for the η/η', $\mathcal{O} = \gamma_5$ while for the δ, $\mathcal{O} = \tau$.

The connected parts of the flavour singlet and corresponding non-singlet meson propagators are the same. In addition to the connected part, the flavour singlet meson propagator has a disconnected part. These are shown schematically in figure 1 where the gluon fields, containing the effects of closed quark loops have been omitted. If this disconnected contribution to the σ and η/η' propagators were to vanish in the plasma phase, then the $U(1)_{axial}$ symmetry would be restored.

When the $SU(2) \times SU(2)$ flavour symmetry is restored,

$$G_\pi = -G_\sigma \tag{2}$$

and

$$G_{\eta'} = -G_\delta \tag{3}$$

If the $U(1)_{axial}$ symmetry were restored,

$$G_\pi = -G_\delta \tag{4}$$

and

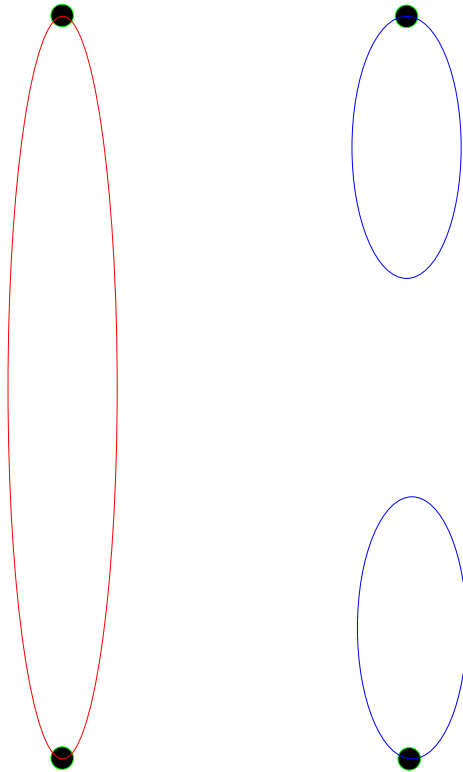$$G_\sigma = -G_{\eta'} \tag{5}$$

in addition.

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Figure 1: Connected (left) and disconnected (right) contributions to meson propagators in a fixed background gauge-field (not shown).

Now let us discuss the rôle of instantons. These arguments are appropriate to the high temperature, quark-gluon plasma phase, where we can take the fermion mass to zero at finite volume, and we expect a gap between zero modes and non-zero modes at mass=0. The spectral decomposition of the quark propagator is

$$S(x, y) = \sum_\lambda \frac{\psi_\lambda(x)\psi_\lambda^\dagger(y)}{i\lambda + m} \tag{6}$$

Now let us return to the consideration of meson propagators. For configurations with no zero modes ($\lambda \neq 0$), only the connected contributions survive the chiral ($m \to 0$) limit. For configurations with one zero mode,

$$S(x, y) \sim 1/m \tag{7}$$

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and the dominant contributions to both the connected and disconnected terms are $\sim 1/m^2$. Such configurations contribute with weight $\propto m^2$ (from the fermion determinant), and so we get a finite contribution. The fermion determinant suppresses contributions from configurations with > 1 zero modes. We find $G_\pi = -G_\sigma$ and $G_{\eta'} = -G_\delta$, i.e. chiral $SU(2) \times SU(2)$ flavour symmetry is restored.

The relationship to topology is expressed by the Atiyah-Singer index theorem[8]:

$$m \int d^4x \mathrm{Tr}\gamma_5 S = n_L - n_R = Q_{top} = \frac{1}{32\pi^2} \int d^4x \mathrm{Tr} F\tilde{F}, \qquad (8)$$

for a given gauge field configuration. This tells us that the topological charge is the difference between the number of left-handed and the number of right-handed zero modes.

A corollary is

$$\chi_{top} = \frac{\langle Q_{top}^2 \rangle}{V} = m^2 \frac{\int d^4x \int d^4y G_{\eta'}^{dis}(x,y)}{V} = m^2 \chi_5^{dis} \qquad (9)$$

where a non-zero value of χ_5^{dis} would indicate that the η/η' propagator differed from that of the π, and that the $U(1)_{axial}$ symmetry was broken.

3 Meson propagators on the lattice.

In lattice QCD, the $SU(2) \times SU(2)$ and $U(1)_{axial}$ symmetries are explicitly broken. We have worked with staggered fermions where 1 generator of $SU(2)_{axial}$ remains unbroken. There are no exact zero modes, and one can only take $m \to 0$ after taking the continuum limit. We must therefore work at small but finite m.

We work with configurations generated in simulations on a $16^3 \times 8$ lattice in lattice QCD with 2 light ($m/T = 0.05$) staggered quark flavours. Most of these were provided by the HTMCGC collaboration[2]. The chiral transition occurs at $5.475 < \beta_c \equiv 6/g_c^2 < 5.4875$.

The meson screening masses have been obtained from exponential fits to propagators for spatial separations. Local operators (noisy point source) were used for the π, σ and δ. Covariant 4-link operators were used for the π_4 and the η'. $U(1)$ noisy estimators were used to extract the disconnected contributions to the σ and η' propagators[9].

Figure 2 shows the meson screening masses obtained from exponential fits to these propagators. (n.b. the flavour non-singlet screening masses, i.e. those from the connected propagators have been reported in[2].) We observe that m_σ
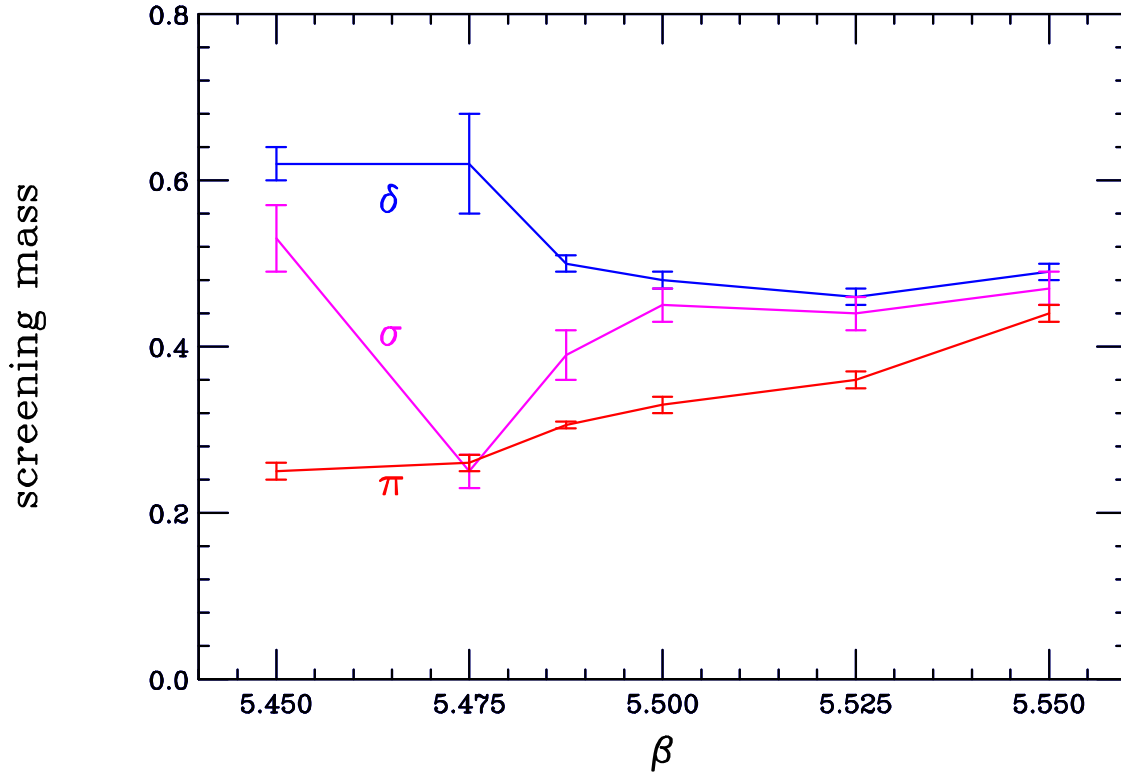
Figure 2: σ, π and δ screening masses as functions of β for full 2-flavour lattice QCD.

and m_δ remain distinct until appreciably above the transition. Since the σ and δ propagators differ only by their disconnected pieces, it is these disconnected pieces which are responsible for this difference. The σ and π masses are only distinct above the transition because of the explicit chiral symmetry breaking from using a finite quark mass.

The scalar disconnected susceptibility (which would be the same as the pseudoscalar in the symmetric phase in the chiral limit, if we had continuum symmetries), shows a clear peak at the transition, and is finite well into the plasma phase.

4 The fate of zero modes on the lattice.[b]

Staggered quarks preserve one $U(1)$ subgroup of the $SU(4)_{axial}$ symmetry. The rest of the $SU(4) \times SU(4)$ chiral symmetry is broken by terms $\mathcal{O}(a^2)$. As in the continuum, the eigenvalues of the Dirac operator have the form:

$$i\lambda + m \tag{10}$$

and occur in complex conjugate pairs. "γ_5" is an operator which involves a displacement by 4 links. It does not anticommute with the $m = 0$ Dirac operator, and does not even have eigenvalues ± 1.

In the continuum,

$$\langle \lambda | \gamma_5 | \lambda \rangle = 0, \quad \lambda \neq 0; \tag{11}$$

and

$$\langle 0 | \gamma_5 | 0 \rangle = \pm 1, \quad \lambda = 0. \tag{12}$$

For staggered quarks this is no longer true. This is fortunate since all modes have $\lambda \neq 0$, except on a set of configurations of measure zero. In order to have a sensible continuum limit, those eigenmodes with λ close to zero should have $\langle \lambda | \gamma_5 | \lambda \rangle$ close to a constant which can be renormalized to ± 1, while other eigenmodes should have $\langle \lambda | \gamma_5 | \lambda \rangle$ close to zero, at sufficiently small coupling. These near-zero eigenmodes should obey

$$n_+ - n_- = Q_{top} \tag{13}$$

where Q_{top} is an estimate of the topological charge of the configuration, determined by other means.

For each of our configurations we have determined the 16 eigenvalues λ which are smallest in magnitude, and their corresponding eigenvectors. We have repeated this determination for a set of quenched configurations on the same size lattice. $\langle \lambda | \gamma_5 | \lambda \rangle$ was calculated for each mode. Q_{top} was measured for each configuration by the cooling method. We have plotted $\langle \lambda | \gamma_5 | \lambda \rangle$ versus λ for the 8 smallest positive eigenvalues, λ for each configuration for a representative β value for our quenched and unquenched simulations in figures 3, 4.

For the quenched case, at $\beta \equiv 6/g^2 = 6.2$ in the high-temperature phase, it is clear that there is a correlation between small eigenvalues and large $\langle \lambda | \gamma_5 | \lambda \rangle$. In addition, we see that such eigenvalues tend to occur for configurations with non-zero topological charge. In the full 2-flavour case at beta=5.55, also in the quark-gluon plasma phase, there is an indication of similar behaviour.

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[b]The discussion at the beginning of this section is condensed from Smit and Vink [10]
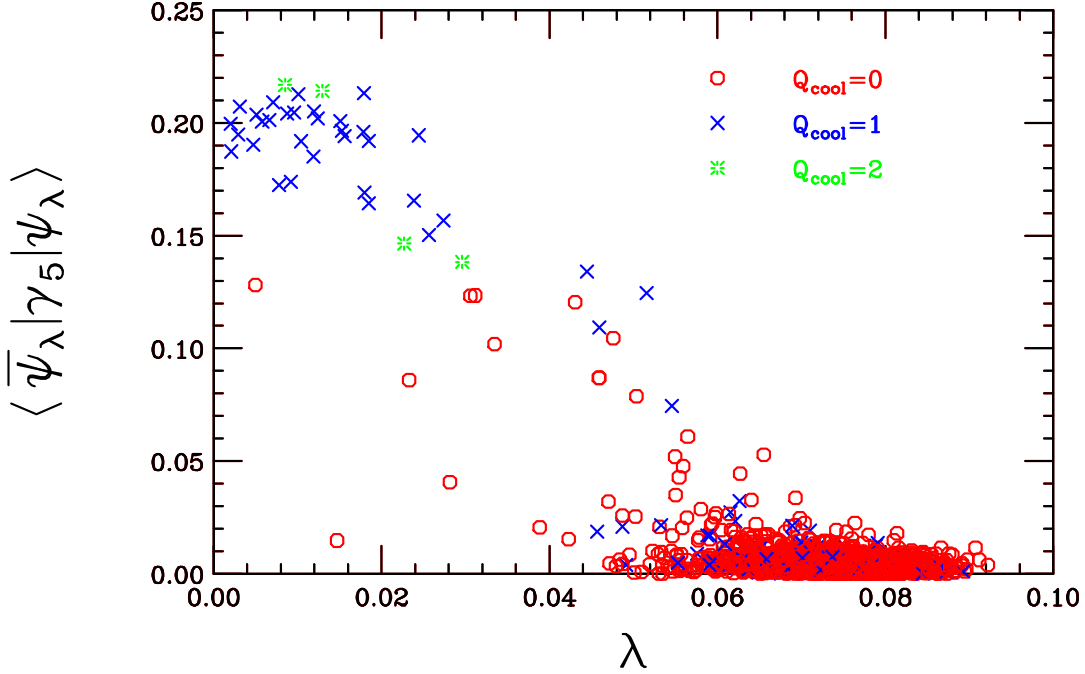
Figure 3: Chirality versus λ at $\beta = 6.2$(quenched QCD).

However, since $\beta = 5.55$ represents a somewhat larger lattice spacing the effects are far less obvious.

We have used these low lying eigenmodes to approximate the quark propagator. Well above the phase transition this approximation gives a very good approximation to the disconnected part of the flavour singlet meson propagators (see figure 5). This supports the conclusion that the disconnected part of these propagators is given entirely by instantons, in the chiral limit.

Because of the finiteness of our would be zero modes, we are forced to remain at finite quark mass. If we continued to zero quark mass $m \int d^4 x \mathrm{Tr}\gamma_5 S$ would vanish, as would the disconnected parts of our singlet meson propagators, leading us to the erroneous conclusion that the $U(1)_{axial}$ symmetry was restored.
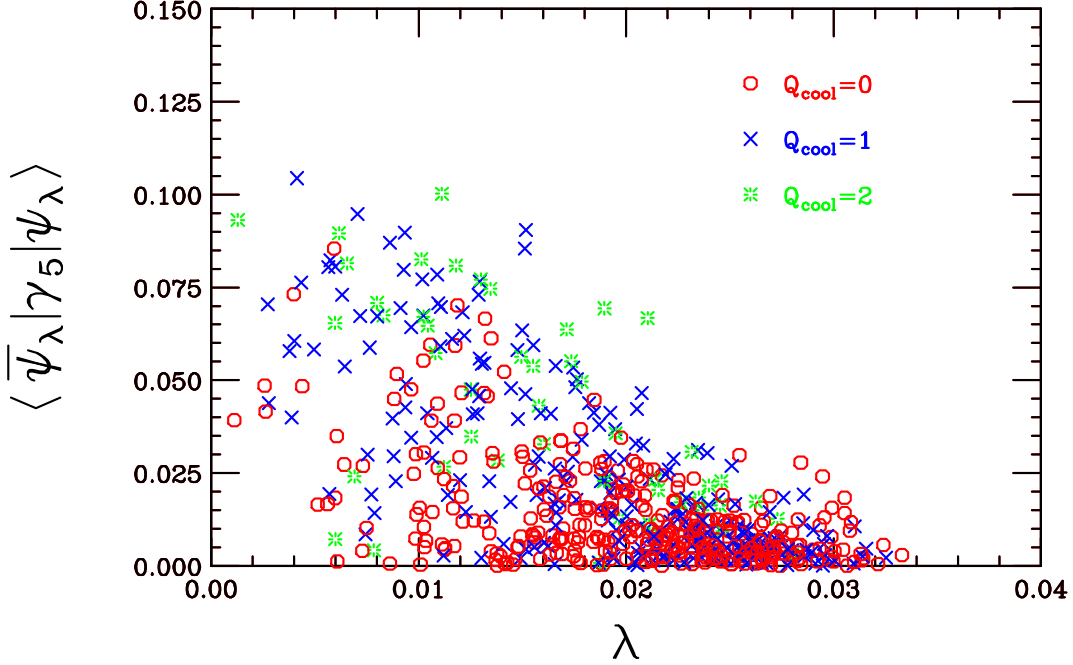
$$N_f=2 \qquad \beta=5.55 \qquad m_q a=0.00625$$

Figure 4: Chirality versus λ at $\beta = 5.55$(full QCD).

5   Better Actions

To get more definitive results would require either using much larger lattices, or using improved actions which better approximate the flavour chiral symmetries of the continuum.

As a first try we have used the "link + staples" action suggested by the MILC collaboration [11] as a way of improving the flavour symmetry of the staggered quark action. This improved the approximation to 4-fold degeneracy of the eigenvalues, which would be present in the continuum. However, the overall improvement in the $\langle\lambda|\gamma_5|\lambda\rangle$ versus λ plot was not great.

We are now investigating how Wilson, and Wilson + Clover Leaf actions perform in this regard on the configurations we have. These actions also have chiral symmetry breaking, but they have exact $SU(2)$ flavour symmetry. The
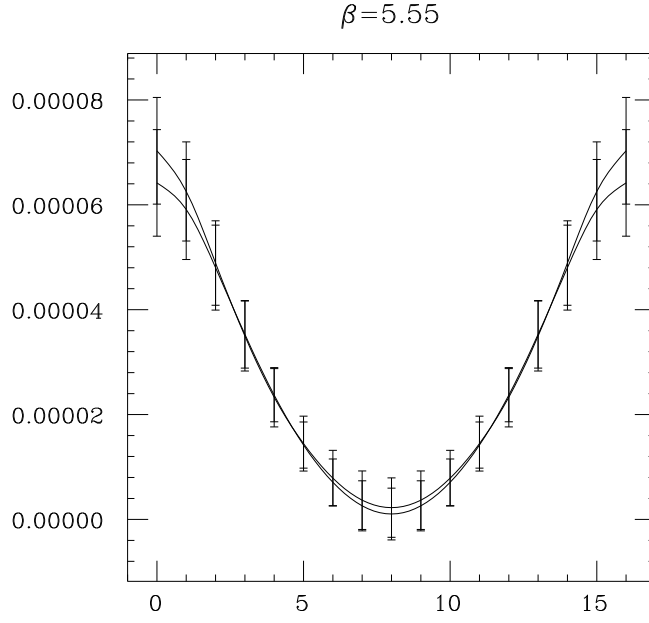
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Figure 5: Comparison of the disconnected pseudoscalar correlators obtained from a noisy estimator (top curve) and from the truncated spectral decomposition of the quark propagator (bottom curve) at $\beta = 5.55$ ($N_f = 2$, $m = 0.00625$).

way which they fail to obey the Atiyah-Singer index theorem is different from the way staggered fermions fail.

Finally, we are investigating how well domain-wall fermions satisfy the Atiyah-Singer index theorem and looking at their zero modes on these configurations. Domain-wall fermions are 5-dimensional Wilson fermions with boundary conditions in the 5th dimension which cause light quark states to reside close to the boundaries in the 5-th dimension[12]. Their advantage is that in the limit of infinite 5-th dimension, chiral $SU(2) \times SU(2)$ flavour symmetry becomes exact, and the approach to this chiral symmetry appears to be exponential in N_5. In this limit, the index theorem should become exact. Our preliminary calculations on a few configurations show promise.

Other improved actions, including ones with gauge improvement should be tried.

6   Discussion and Conclusions

At the phase transition from hadronic matter to a quark-gluon plasma in QCD with 2 zero mass quarks, the chiral $SU(2) \times SU(2)$ chiral flavour symmetry is restored. We have found evidence that the anomalous $U(1)_{axial}$ symmetry is not restored, but remains broken in the plasma phase. The explicit breaking of chiral symmetry by the staggered lattice fermions is sufficiently large at these lattice spacings to somewhat obscure these results, as does the explicit chiral symmetry breaking due to our use of finite quark masses. We are prevented from decreasing the quark masses to zero by this chiral symmetry breaking which renders would-be zero modes non-zero. Since our analytic arguments required a fixed spatial volume, we need to repeat our simulations on several spatial volumes to study finite size effects, to see that our conclusions survive in the infinite volume limit.

One can improve on our results by setting the $4Q_{top}$ eigenvalues of smallest magnitude to zero by hand. This gives us a non-zero $m \int d^4x \text{Tr} \gamma_5 S$ and non-zero disconnected contributions to our singlet meson propagators. However, the chiral flavour symmetry breaking associated with staggered fermions manifests itself in ways other than the zero mode shift, and these are not addressed by this procedure. To address these requires us to study more systematic improvements to finite temperature simulations and measurements involving light quarks.

If the $U(1)_{axial}$ symmetry remains broken in the high temperature phase, this means that instantons continue to play an important rôle in this phase. In particular they contribute the disconnected part of the propagator which is responsible for the mass difference between isosinglet and isotriplet scalar and pseudoscalar mesonic excitations of the plasma. In addition, they make contributions to the connected meson propagators. This occurs because, despite the fact that the fermion determinant vanishes as m^2 in the chiral limit for the $Q_{top} = \pm 1$ sector, the connected and disconnected contributions to the meson propagators for gauge configurations in this sector diverge as $1/m^2$ giving a net finite contribution. Thus, while $m_\pi = m_\sigma$ and $m_{\eta'} = m_\delta$, $m_\pi \neq m_{\eta'}$ and $m_\sigma \neq m_\delta$. It will be interesting to see if the meson propagators in the $Q_{top} = 0$ sector yield screening masses close to $2\pi T$, the screening mass for a free quark-antiquark pair. If so, it would indicate that it is the instantons which prevent the quark-gluon plasma from being described as a gas of free quarks and gluons.

It is important that the chiral flavour symmetry is $SU(2) \times SU(2)$. For $SU(3) \times SU(3)$ chiral flavour symmetry, instantons are further suppressed in the plasma phase, and the $U(1)_{axial}$ symmetry is restored at the transition, which now becomes first order.

At high temperatures, the disconnected contributions to the flavour singlet meson propagators are well approximated by the contributions of a few low lying modes of the quark propagator.

We need improved actions — especially fermion actions — to get more definitive results, unless we have the computing power to work with the much larger lattices needed to enable us to decrease the lattice spacing enough to significantly reduce the lattice breaking of chiral flavour symmetry. Domain-wall fermions appear to be a good bet. It is of interest to know if such improvements also give one better critical indices at the phase transition. Others have suggested that the "perfect action" improvements are the most promising [13].

Even with the ultimate improved action, currently used simulation methods will fair poorly when used to calculate meson propagators in the chiral limit of the plasma phase. This is because, as mentioned above, configurations which occur with probability only $\mathcal{O}(m^2)$, each contribute a term $\mathcal{O}(1/m^2)$ to the meson propagators. Hence, in the chiral limit, important, and in fact infinite contributions come from configurations which never occur. We are studying algorithm modifications to better accommodate this limit.

Acknowledgments

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